Answers of HW Questions

Chap 1: Mathematical Premiminaries

1.2

3. The largest intervals are **a.** (149.85,150.15) **b.** (899.1, 900.9) **c.** (1498.5, 1501.5) **d.** (89.91,90.09)

11. a. $\lim_{x\to 0} \frac{x\cos x - \sin x}{x - \sin x} = \lim_{x\to 0} \frac{-x\sin x}{1-\cos x} = \lim_{x\to 0} \frac{-\sin x - x\cos x}{\sin x} = \lim_{x\to 0} \frac{-2\cos x + x\sin x}{\cos x} = -2$

b. -1.941

c. $\frac{x(1-\frac{1}{2}x^2) - (x - \frac{1}{6}x^3)}{x - (x - \frac{1}{6}x^3)} = -2$

d. The relative error in part (b) is 0.029. The relative error in part (c) is 0.00050.

17. b. The first formula gives -0.00658, and the second formula gives -0.0100. The true three-digit value is -0.0116.

1.3

7. The rates of convergence are:
a. $O(h^2)$ **b.** $O(h)$ **c.** $O(h^2)$ **d.** $O(h)$

Chap 2: Solutions of Equations in One Variable

2.1

13. A bound is $n \geq 14$, and $p_{14} = 1.32477$.

15. Since $\lim_{n\to\infty}(p_n - p_{n-1}) = \lim_{n\to\infty} 1/n = 0$, the difference in the terms goes to zero. However, p_n is the nth term of the divergent harmonic series, so $\lim_{n\to\infty} p_n = \infty$.

2.2

3. The order in descending speed of convergence is (b), (d), (a). The sequence in (c) does not converge.

第 19 题无参考答案。

2.4

11. If $\frac{|p_{n+1}-p|}{|p_n-p|^3} = 0.75$ and $|p_0 - p| = 0.5$, then

$$|p_n - p| = (0.75)^{(3^n-1)/2}|p_0 - p|^{3^n}.$$

To have $|p_n - p| \leq 10^{-8}$ requires that $n \geq 3$.

Chap 3: Interpolation and Polynomial Approximation

3.1

5. $\sqrt{3} \approx P_4\left(\frac{1}{2}\right) = 1.708\overline{3}$

17. The largest possible step size is 0.004291932, so 0.04 would be a reasonable choice.

3.2

5. **a.** $f(0.05) \approx 1.05126$ **b.** $f(0.65) \approx 1.91555$ **c.** $f(0.43) \approx 1.53725$

13. $f[x_0] = f(x_0) = 1$, $f[x_1] = f(x_1) = 3$, $f[x_0, x_1] = 5$

3.3

7. The Hermite polynomial generated from these data is

$$\begin{aligned} H_9(x) = {} & 75x + 0.222222x^2(x-3) - 0.0311111x^2(x-3)^2 - 0.00644444x^2(x-3)^2(x-5) \\ & + 0.00226389x^2(x-3)^2(x-5)^2 - 0.000913194x^2(x-3)^2(x-5)^2(x-8) \\ & + 0.000130527x^2(x-3)^2(x-5)^2(x-8)^2 - 0.0000202236x^2(x-3)^2(x-5)^2(x-8)^2(x-13). \end{aligned}$$

a. The Hermite polynomial predicts a position of $H_9(10) = 743$ ft and a speed of $H_9'(10) = 48$ ft/s. Although the position approximation is reasonable, the low speed prediction is suspect.

b. To find the first time the speed exceeds 55 mi/h $= 80.\overline{6}$ ft/s, we solve for the smallest value of t in the equation $80.\overline{6} = H_9'(x)$. This gives $x \approx 5.6488092$.

c. The estimated maximum speed is $H_9'(12.37187) = 119.423$ ft/s ≈ 81.425 mi/h.

3.4

9. $B = \frac{1}{4}$, $D = \frac{1}{4}$, $b = -\frac{1}{2}$, $d = \frac{1}{4}$

17. The piecewise linear approximation to f is given by

$$F(x) = \begin{cases} 20(e^{0.1} - 1)x + 1, & \text{for } x \text{ in } [0, 0.05] \\ 20(e^{0.2} - e^{0.1})x + 2e^{0.1} - e^{0.2}, & \text{for } x \text{ in } (0.05, 1]. \end{cases}$$

We have

$$\int_0^{0.1} F(x)\,dx = 0.1107936 \quad \text{and} \quad \int_0^{0.1} f(x)\,dx = 0.1107014.$$

Chap 4: Numerical Differentiation and Integration

4.1

7. $f'(3) \approx \frac{1}{12}[f(1) - 8f(2) + 8f(4) - f(5)] = 0.21062$, with an error bound given by

$$\max_{1 \le x \le 5} \frac{|f^{(5)}(x)|h^4}{30} \le \frac{23}{30} = 0.7\overline{6}.$$

13. The approximation is -4.8×10^{-9}. $f''(0.5) = 0$. The error bound is 0.35874. The method is very accurate since the function is symmetric about $x = 0.5$.

4.3

7. $f(1) = \frac{1}{2}$
9. The degree of precision is 3.
11. $c_0 = \frac{1}{3}$, $c_1 = \frac{4}{3}$, $c_2 = \frac{1}{3}$
13. $c_0 = c_1 = \frac{1}{2}$ gives the highest degree of precision, 1.

4.4

7. **a.** The Composite Trapezoidal rule requires $h < 0.000922295$ and $n \ge 2168$.
   b. The Composite Simpson's rule requires $h < 0.037658$ and $n \ge 54$.
   c. The Composite Midpoint rule requires $h < 0.00065216$ and $n \ge 3066$.

4.7

1. Gaussian quadrature gives: **a.** 0.1922687 **b.** 0.1594104 **c.** −0.1768190 **d.** 0.08926302 **e.** 2.5913247 **f.** −0.7307230 **g.** 0.6361966 **h.** 0.6423172

5. $a = 1$, $b = 1$, $c = \frac{1}{3}$, $d = -\frac{1}{3}$

Chap 5: Initial-Value Problems for Ordinary Differential Equations

5.3

5. **a.** Taylor's method of order two gives the results in the following table.

i	t_i	w_i	$y(t_i)$
1	1.1	0.3397852	0.3459199
5	1.5	3.910985	3.967666
6	1.6	5.643081	5.720962
9	1.9	14.15268	14.32308
10	2.0	18.46999	18.68310

   b. Linear interpolation gives $y(1.04) \approx 0.1359139$, $y(1.55) \approx 4.777033$, and $y(1.97) \approx 17.17480$. Actual values are $y(1.04) = 0.1199875$, $y(1.55) = 4.788635$, and $y(1.97) = 17.27930$.

5.4

1. a.

t	Modified Euler	$y(t)$
0.5	0.5602111	0.2836165
1.0	5.3014898	3.2190993

b.

t	Modified Euler	$y(t)$
2.5	1.8125000	1.8333333
3.0	2.4815531	2.5000000

c.

t	Modified Euler	$y(t)$
1.25	2.7750000	2.7789294
1.50	3.6008333	3.6081977
1.75	4.4688294	4.4793276
2.00	5.3728586	5.3862944

d.

t	Modified Euler	$y(t)$
0.25	1.3199027	1.3291498
0.50	1.7070300	1.7304898
0.75	2.0053560	2.0414720
1.00	2.0770789	2.1179795

第 10 和 13 题均无参考答案。

5.6

第 10 题无参考答案。

5.9

5. The Adams fourth-order predictor-corrector method for systems gives the results in the following tables.

a.

t_i	w_{1i}	$y(t_i)$
0.200	0.00015352	0.00015350
0.500	0.00743133	0.00743027
0.700	0.03300266	0.03299805
1.000	0.17134711	0.17132880

b.

t_i	w_{1i}	$y(t_i)$
1.200	0.96152437	0.96152583
1.500	0.77796798	0.77797237
1.700	0.59373213	0.59373830
2.000	0.27258055	0.27258872

c.

t_i	w_{1i}	$y(t_i)$
1.000	3.73186337	3.73170445
2.000	11.31462595	11.31452924
3.000	34.04548233	34.04517155

d.

t_i	w_{1i}	$y(t_i)$
1.200	0.27273759	0.27273791
1.500	1.08847933	1.08849259
1.700	2.04352376	2.04353642
2.000	4.36157310	4.36157780

5.10

7. The method is unstable.

Chap 6: Direct Methods for Solving Linear Systems

6.1

第 8 题无参考答案。

11. b. The results for this exercise are listed in the following table. (The abbreviations M/D and A/S are used for multiplications/divisions and additions/subtractions, respectively.)

	Gaussian Elimination		Gauss-Jordan	
n	M/D	A/S	M/D	A/S
3	17	11	21	12
10	430	375	595	495
50	44150	42875	64975	62475
100	343300	338250	509950	499950

6.5

7. c.

	Multiplications/Divisions	Additions/Subtractions
Factoring into LU	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly = b$	$\frac{1}{2}n^2 - \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Solving $Ux = y$	$\frac{1}{2}n^2 + \frac{1}{2}n$	$\frac{1}{2}n^2 - \frac{1}{2}n$
Total	$\frac{1}{3}n^3 + n^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n$

d.

	Multiplications/Divisions	Additions/Subtractions
Factoring into LU	$\frac{1}{3}n^3 - \frac{1}{3}n$	$\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$
Solving $Ly^{(k)} = b^{(k)}$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Solving $Ux^{(k)} = y^{(k)}$	$(\frac{1}{2}n^2 + \frac{1}{2}n)m$	$(\frac{1}{2}n^2 - \frac{1}{2}n)m$
Total	$\frac{1}{3}n^3 + mn^2 - \frac{1}{3}n$	$\frac{1}{3}n^3 + (m - \frac{1}{2})n^2 - (m - \frac{1}{6})n$

（只有 c, d 两小问的参考答案）

6.6

17. a. Since $\det A = 3\alpha - 2\beta$, A is singular if and only if $\alpha = 2\beta/3$. **b.** $|\alpha| > 1, |\beta| < 1$ **c.** $\beta = 1$ **d.** $\alpha > \frac{2}{3}, \beta = 1$

Chap 7: Iterative Techniques in Matrix Algebra

7.1

5. a. We have $||\mathbf{x}-\hat{\mathbf{x}}||_\infty = 8.57 \times 10^{-4}$ and $||A\hat{\mathbf{x}}-\mathbf{b}||_\infty = 2.06 \times 10^{-4}$.
   b. We have $||\mathbf{x}-\hat{\mathbf{x}}||_\infty = 0.90$ and $||A\hat{\mathbf{x}}-\mathbf{b}||_\infty = 0.27$.
   c. We have $||\mathbf{x}-\hat{\mathbf{x}}||_\infty = 0.5$ and $||A\hat{\mathbf{x}}-\mathbf{b}||_\infty = 0.3$.
   d. We have $||\mathbf{x}-\hat{\mathbf{x}}||_\infty = 6.55 \times 10^{-2}$, and $||A\hat{\mathbf{x}}-\mathbf{b}||_\infty = 0.32$.

7. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Then $\|AB\|_\otimes = 2$, but $\|A\|_\otimes \cdot \|B\|_\otimes = 1$.

第 13 题无参考答案。

7.2

3. Only the matrix in (c) is convergent.

7.3

第 13 题无参考答案。

7.4

1. The $\|\cdot\|_\infty$ condition number is: a. 50 b. 241.37 c. 600.002 d. 339,866 e. 12 h. 198.17

9. For the 3×3 Hilbert matrix H, we have

$$\hat{H}^{-1} = \begin{bmatrix} 8.968 & -35.77 & 29.77 \\ -35.77 & 190.6 & -178.6 \\ 29.77 & -178.6 & 178.6 \end{bmatrix}, \qquad \hat{H} = \begin{bmatrix} 0.9799 & 0.4870 & 0.3238 \\ 0.4860 & 0.3246 & 0.2434 \\ 0.3232 & 0.2433 & 0.1949 \end{bmatrix},$$

and $\|H - \hat{H}\|_\infty = 0.04260$.

Chap 8: Approximation Theory

8.1

5. a. The linear least-squares polynomial is $72.0845x - 194.138$, with error 329.
   b. The least-squares polynomial of degree two is $6.61821x^2 - 1.14352x + 1.23556$, with error 1.44×10^{-3}.
   c. The least-squares polynomial of degree three is $-0.0136742x^3 + 6.84557x^2 - 2.37919x + 3.42904$, with error 5.27×10^{-4}.
   d. The least-squares approximation of the form be^{ax} is $24.2588e^{0.372382x}$, with error 418.
   e. The least-squares approximation of the form bx^a is $6.23903x^{2.01954}$, with error 0.00703.

8.2

3. The linear least-squares approximations on $[-1, 1]$ are:

a. $P_1(x) = 3.333333 - 2x$

b. $P_1(x) = 0.6000025x$

c. $P_1(x) = 0.5493063 - 0.2958375x$

d. $P_1(x) = 1.175201 + 1.103639x$

e. $P_1(x) = 0.4207355 + 0.4353975x$

f. $P_1(x) = 0.6479184 - 0.5281226x$

11. The Laguerre polynomials are $L_1(x) = x - 1$, $L_2(x) = x^2 - 4x + 2$ and $L_3(x) = x^3 - 9x^2 + 18x - 6$.

8.3

3. The interpolating polynomials of degree three are:

a. $P_3(x) = 2.519044 + 1.945377(x - 0.9238795) + 0.7047420(x - 0.9238795)(x - 0.3826834)$
$+ 0.1751757(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

b. $P_3(x) = 0.7979459 + 0.7844380(x - 0.9238795) - 0.1464394(x - 0.9238795)(x - 0.3826834)$
$- 0.1585049(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

c. $P_3(x) = 1.072911 + 0.3782067(x - 0.9238795) - 0.09799213(x - 0.9238795)(x - 0.3826834)$
$+ 0.04909073(x - 0.9238795)(x - 0.3826834)(x + 0.3826834)$

d. $P_3(x) = 0.7285533 + 1.306563(x - 0.9238795) + 0.9999999(x - 0.9238795)(x - 0.3826834)$

7. The cubic polynomial $\frac{383}{384}x - \frac{5}{32}x^3$ approximates $\sin x$ with error at most 7.19×10^{-4}.

9. The change of variable $x = \cos\theta$ produces

$$\int_{-1}^{1} \frac{T_n^2(x)}{\sqrt{1-x^2}}\,dx = \int_{-1}^{1} \frac{[\cos(n \arccos x)]^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi} (\cos(n\theta))^2\,dx = \frac{\pi}{2}.$$

Chap 9: Approximating Eigenvalues

无