Collection of HW Questions

Chap 1: Mathematical Premiminaries

1.2

3. Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for each value of p.

    a. 150   **b.** 900   **c.** 1500   **d.** 90

11. Let

$$f(x) = \frac{x \cos x - \sin x}{x - \sin x}.$$

    a. Find $\lim_{x \to 0} f(x)$.

    b. Use four-digit rounding arithmetic to evaluate $f(0.1)$.

    c. Replace each trigonometric function with its third Maclaurin polynomial, and repeat part (b).

    d. The actual value is $f(0.1) = -1.99899998$. Find the relative error for the values obtained in parts (b) and (c).

17. Suppose two points (x_0, y_0) and (x_1, y_1) are on a straight line with $y_1 \neq y_0$. Two formulas are available to find the x-intercept of the line:

$$x = \frac{x_0 y_1 - x_1 y_0}{y_1 - y_0} \quad \text{and} \quad x = x_0 - \frac{(x_1 - x_0) y_0}{y_1 - y_0}.$$

    b. Use the data $(x_0, y_0) = (1.31, 3.24)$ and $(x_1, y_1) = (1.93, 4.76)$ and three-digit rounding arithmetic to compute the x-intercept both ways. Which method is better and why?

1.3

7. Find the rates of convergence of the following functions as $h \to 0$.

    a. $\lim_{h \to 0} \dfrac{\sin h}{h} = 1$ **b.** $\lim_{h \to 0} \dfrac{1 - \cos h}{h} = 0$

    c. $\lim_{h \to 0} \dfrac{\sin h - h \cos h}{h} = 0$ **d.** $\lim_{h \to 0} \dfrac{1 - e^h}{h} = -1$

Chap 2: Solutions of Equations in One Variable

2.1

13. Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4} to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 2]$. Find an approximation to the root with this degree of accuracy.

Theorem 2.1   Suppose that $f \in C[a, b]$ and $f(a) \cdot f(b) < 0$. The Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with

$$|p_n - p| \le \frac{b-a}{2^n}, \quad \text{when} \quad n \ge 1.$$ ∎

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15. Let $\{p_n\}$ be the sequence defined by $p_n = \sum_{k=1}^{n} \frac{1}{k}$. Show that $\{p_n\}$ diverges even though $\lim_{n \to \infty}(p_n - p_{n-1}) = 0$.

2.2

3. The following four methods are proposed to compute $21^{1/3}$. Rank them in order, based on their apparent speed of convergence, assuming $p_0 = 1$.

   a. $p_n = \dfrac{20p_{n-1} + 21/p_{n-1}^2}{21}$ **b.** $p_n = p_{n-1} - \dfrac{p_{n-1}^3 - 21}{3p_{n-1}^2}$

   c. $p_n = p_{n-1} - \dfrac{p_{n-1}^4 - 21p_{n-1}}{p_{n-1}^2 - 21}$ **d.** $p_n = \left(\dfrac{21}{p_{n-1}}\right)^{1/2}$

19.   **a.**   Use Theorem 2.3 to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \ge 1,$$

converges to $\sqrt{2}$ whenever $x_0 > \sqrt{2}$.

   b.   Use the fact that $0 < (x_0 - \sqrt{2})^2$ whenever $x_0 \ne \sqrt{2}$ to show that if $0 < x_0 < \sqrt{2}$, then $x_1 > \sqrt{2}$.

   c.   Use the results of parts (a) and (b) to show that the sequence in (a) converges to $\sqrt{2}$ whenever $x_0 > 0$.

Theorem 2.3   **(Fixed-Point Theorem)**

Let $g \in C[a, b]$ be such that $g(x) \in [a, b]$, for all x in $[a, b]$. Suppose, in addition, that g' exists on (a, b) and that a constant $0 < k < 1$ exists with

$$|g'(x)| \le k, \quad \text{for all } x \in (a, b).$$

Then, for any number p_0 in $[a, b]$, the sequence defined by

$$p_n = g(p_{n-1}), \quad n \ge 1,$$

converges to the unique fixed point p in $[a, b]$.   ∎

2.4

11. The iterative method to solve $f(x) = 0$, given by the fixed-point method $g(x) = x$, where

$$p_n = g(p_{n-1}) = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} - \frac{f''(p_{n-1})}{2f'(p_{n-1})} \left[\frac{f(p_{n-1})}{f'(p_{n-1})} \right]^2, \quad \text{for } n = 1, 2, 3, \ldots.$$

has $g'(p) = g''(p) = 0$. This will generally yield cubic ($\alpha = 3$) convergence. Expand the analysis of Example 1 to compare quadratic and cubic convergence.

注：Example 1 在教材 P_{79}

Chap 3: Interpolation and Polynomial Approximation

3.1

5. Use Neville's method to approximate $\sqrt{3}$ with the function $f(x) = 3^x$ and the values $x_0 = -2$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 2$.

17. Suppose you need to construct eight-decimal-place tables for the common, or base-10, logarithm function from $x = 1$ to $x = 10$ in such a way that linear interpolation is accurate to within 10^{-6}. Determine a bound for the step size for this table. What choice of step size would you make to ensure that $x = 10$ is included in the table?

3.2

5. a. Approximate $f(0.05)$ using the following data and the Newton forward divided-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

b. Use the Newton backward divided-difference formula to approximate $f(0.65)$.

13. For a function f, the forward divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table.

3.3

7. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Time	0	3	5	8	13
Distance	0	225	383	623	993
Speed	75	77	80	74	72

   a. Use a Hermite polynomial to predict the position of the car and its speed when $t = 10$ s.
   b. Use the derivative of the Hermite polynomial to determine whether the car ever exceeds a 55 mi/h speed limit on the road. If so, what is the first time the car exceeds this speed?
   c. What is the predicted maximum speed for the car?

3.4

9. A natural cubic spline S is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + B(x-1) - D(x-1)^3, & \text{if } 1 \le x < 2. \\ S_1(x) = 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3, & \text{if } 2 \le x \le 3. \end{cases}$$

If S interpolates the data $(1, 1)$, $(2, 1)$, and $(3, 0)$, find B, D, b, and d.

17. Given the partition $x_0 = 0$, $x_1 = 0.05$, and $x_2 = 0.1$ of $[0, 0.1]$, find the piecewise linear interpolating function F for $f(x) = e^{2x}$. Approximate $\int_0^{0.1} e^{2x}\, dx$ with $\int_0^{0.1} F(x)\, dx$, and compare the results to the actual value.

Chap 4: Numerical Differentiation and Integration

4.1

7. Use the following data and the knowledge that the first five derivatives of f are bounded on $[1, 5]$ by 2, 3, 6, 12 and 23, respectively, to approximate $f'(3)$ as accurately as possible. Find a bound for the error.

x	1	2	3	4	5
$f(x)$	2.4142	2.6734	2.8974	3.0976	3.2804

13. Let $f(x) = \cos \pi x$. Use Eq. (4.9) and the values of $f(x)$ at $x = 0.25$, 0.5, and 0.75 to approximate $f''(0.5)$. Compare this result to the exact value and to the approximation found in Exercise 11 of Section 3.4. Explain why this method is particularly accurate for this problem, and find a bound for the error.

$$f''(x_0) = \frac{1}{h^2}[f(x_0 - h) - 2f(x_0) + f(x_0 + h)] - \frac{h^2}{12}f^{(4)}(\xi). \qquad (4.9)$$

11. Construct a free cubic spline to approximate $f(x) = \cos \pi x$ by using the values given by $f(x)$ at $x = 0, 0.25, 0.5, 0.75$, and 1.0. Integrate the spline over $[0, 1]$, and compare the result to $\int_0^1 \cos \pi x \, dx = 0$. Use the derivatives of the spline to approximate $f'(0.5)$ and $f''(0.5)$. Compare these approximations to the actual values.

4.3

7. The Trapezoidal rule applied to $\int_0^2 f(x) \, dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

9. Find the degree of precision of the quadrature formula

$$\int_{-1}^1 f(x) \, dx = f\left(-\frac{\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right).$$

11. The quadrature formula $\int_{-1}^1 f(x) \, dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1, and c_2.

13. Find the constants c_0, c_1, and x_1 so that the quadrature formula

$$\int_0^1 f(x) \, dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

4.4

7. Determine the values of n and h required to approximate

$$\int_0^2 e^{2x} \sin 3x \, dx$$

to within 10^{-4}.
   a. Use the Composite Trapezoidal rule.
   b. Use the Composite Simpson's rule.

4.7

1. Approximate the following integrals using Gaussian quadrature with $n = 2$, and compare your results to the exact values of the integrals.

    a. $\int_1^{1.5} x^2 \ln x \, dx$

    b. $\int_0^1 x^2 e^{-x} \, dx$

    c. $\int_0^{0.35} \frac{2}{x^2 - 4} \, dx$

    d. $\int_0^{\pi/4} x^2 \sin x \, dx$

    e. $\int_0^{\pi/4} e^{3x} \sin 2x \, dx$

    f. $\int_1^{1.6} \frac{2x}{x^2 - 4} \, dx$

    g. $\int_3^{3.5} \frac{x}{\sqrt{x^2 - 4}} \, dx$

    h. $\int_0^{\pi/4} (\cos x)^2 \, dx$

5. Determine constants a, b, c, and d that will produce a quadrature formula

    $$\int_{-1}^{1} f(x) \, dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

    that has degree of precision 3.

Chap 5: Initial-Value Problems for Ordinary Differential Equations

5.3

5. Given the initial-value problem

    $$y' = \frac{2}{t} y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = 0,$$

    with exact solution $y(t) = t^2(e^t - e)$:

    a. Use Taylor's method of order two with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

    b. Use the answers generated in part (a) and linear interpolation to approximate y at the following values, and compare them to the actual values of y.

    i. $y(1.04)$ ii. ' $y(1.55)$ iii. $y(1.97)$

5.4

1. Use the Modified Euler method to approximate the solutions to each of the following initial-value problems, and compare the results to the actual values.

    a. $y' = te^{3t} - 2y$, $\quad 0 \le t \le 1$, $\quad y(0) = 0$, with $h = 0.5$; actual solution $y(t) = \frac{1}{5}te^{3t} - \frac{1}{25}e^{3t} + \frac{1}{25}e^{-2t}$.

    b. $y' = 1 + (t-y)^2$, $\quad 2 \le t \le 3$, $\quad y(2) = 1$, with $h = 0.5$; actual solution $y(t) = t + \frac{1}{1-t}$.

    c. $y' = 1 + y/t$, $\quad 1 \le t \le 2$, $\quad y(1) = 2$, with $h = 0.25$; actual solution $y(t) = t \ln t + 2t$.

    d. $y' = \cos 2t + \sin 3t$, $\quad 0 \le t \le 1$, $\quad y(0) = 1$, with $h = 0.25$; actual solution $y(t) = \frac{1}{2}\sin 2t - \frac{1}{3}\cos 3t + \frac{4}{3}$.

10. Repeat Exercise 1 using the Runge-Kutta method of order four.

13. Show that the Midpoint method, the Modified Euler method, and Heun's method give the same approximations to the initial-value problem

$$y' = -y + t + 1, \quad 0 \le t \le 1, \quad y(0) = 1,$$

    for any choice of h. Why is this true?

5.6

10. Derive Eq. (5.33) by the following method. Set

$$y(t_{i+1}) = y(t_i) + ahf(t_i, y(t_i)) + bhf(t_{i-1}, y(t_{i-1})) + chf(t_{i-2}, y(t_{i-2})).$$

    Expand $y(t_{i+1})$, $f(t_{i-2}, y(t_{i-2}))$, and $f(t_{i-1}, y(t_{i-1}))$ in Taylor series about $(t_i, y(t_i))$, and equate the coefficients of h, h^2 and h^3 to obtain a, b, and c.

Adams-Bashforth Three-Step Explicit Method:

$$w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2,$$

$$w_{i+1} = w_i + \frac{h}{12}[23f(t_i, w_i) - 16f(t_{i-1}, w_{i-1}) + 5f(t_{i-2}, w_{i-2})], \qquad (5.33)$$

where $i = 2, 3, \ldots, N - 1$. The local truncation error is $\tau_{i+1}(h) = \frac{3}{8}y^{(4)}(\mu_i)h^3$, for some $\mu_i \in (t_{i-2}, t_{i+1})$.

5.9

5. Repeat Exercise 2 using the algorithm developed in Exercise 3.

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2. Use the Runge-Kutta for Systems Algorithm to approximate the solutions of the following higher-order differential equations, and compare the results to the actual solutions.

    a. $y'' - 2y' + y = te^t - t$, $\quad 0 \le t \le 1$, $\quad y(0) = y'(0) = 0$, with $h = 0.1$; actual solution $y(t) = \frac{1}{6}t^3e^t - te^t + 2e^t - t - 2$.

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b. $t^2 y'' - 2ty' + 2y = t^3 \ln t$, $\quad 1 \le t \le 2$, $\quad y(1) = 1$, $\quad y'(1) = 0$, with $h = 0.1$; actual solution $y(t) = \frac{7}{4}t + \frac{1}{2}t^3 \ln t - \frac{3}{4}t^3$.

c. $y''' + 2y'' - y' - 2y = e^t$, $\quad 0 \le t \le 3$, $\quad y(0) = 1$, $\quad y'(0) = 2$, $\quad y''(0) = 0$, with $h = 0.2$; actual solution $y(t) = \frac{43}{36}e^t + \frac{1}{4}e^{-t} - \frac{4}{9}e^{-2t} + \frac{1}{6}te^t$.

d. $t^3 y''' - t^2 y'' + 3ty' - 4y = 5t^3 \ln t + 9t^3$, $\quad 1 \le t \le 2$, $\quad y(1) = 0$, $\quad y'(1) = 1$, $\quad y''(1) = 3$, with $h = 0.1$; actual solution $y(t) = -t^2 + t\cos(\ln t) + t\sin(\ln t) + t^3 \ln t$.

3. Change the Adams Fourth-Order Predictor-Corrector Algorithm to obtain approximate solutions to systems of first-order equations.

5.10

7. Investigate stability for the difference method

$$w_{i+1} = -4w_i + 5w_{i-1} + 2h[f(t_i, w_i) + 2hf(t_{i-1}, w_{i-1})],$$

for $i = 1, 2, \dots, N-1$, with starting values w_0, w_1.

Chap 6: Direct Methods for Solving Linear Systems

6.1

8. Gauss-Jordan Method: This method is described as follows. Use the ith equation to eliminate not only x_i from the equations $E_{i+1}, E_{i+2}, \dots, E_n$, as was done in the Gaussian elimination method, but also from $E_1, E_2, \dots, E_{i-1}$. Upon reducing $[A, \mathbf{b}]$ to:

$$\begin{bmatrix} a_{11}^{(1)} & 0 & \cdots & 0 & \vdots & a_{1,n+1}^{(1)} \\ 0 & a_{22}^{(2)} & \ddots & \vdots & \vdots & a_{2,n-1}^{(2)} \\ \vdots & \ddots & \ddots & 0 & \vdots & \vdots \\ 0 & \cdots & 0 & a_{nn}^{(n)} & \vdots & a_{n,n+1}^{(n)} \end{bmatrix},$$

the solution is obtained by setting

$$x_i = \frac{a_{i,n+1}^{(i)}}{a_{ii}^{(i)}},$$

for each $i = 1, 2, \dots, n$. This procedure circumvents the backward substitution in the Gaussian elimination. Construct an algorithm for the Gauss-Jordan procedure patterned after that of Algorithm 6.1.

11. a. Show that the Gauss-Jordan method requires

$$\frac{n^3}{2} + n^2 - \frac{n}{2} \quad \text{multiplications/divisions}$$

and

$$\frac{n^3}{2} - \frac{n}{2} \quad \text{additions/subtractions.}$$

b. Make a table comparing the required operations for the Gauss-Jordan and Gaussian elimination methods for $n = 3, 10, 50, 100$. Which method requires less computation?

6.5

7. a. Show that the LU Factorization Algorithm requires $\frac{1}{3}n^3 - \frac{1}{3}n$ multiplications/divisions and $\frac{1}{3}n^3 - \frac{1}{2}n^2 + \frac{1}{6}n$ additions/subtractions.

b. Show that solving $Ly = b$, where L is a lower-triangular matrix with $l_{ii} = 1$ for all i, requires $\frac{1}{2}n^2 - \frac{1}{2}n$ multiplications/divisions and $\frac{1}{2}n^2 - \frac{1}{2}n$ additions/subtractions.

c. Show that solving $Ax = b$ by first factoring A into $A = LU$ and then solving $Ly = b$ and $Ux = y$ requires the same number of operations as the Gaussian Elimination Algorithm 6.1.

d. Count the number of operations required to solve m linear systems $Ax^{(k)} = b^{(k)}$ for $k = 1, \ldots, m$ by first factoring A and then using the method of part (c) m times.

6.6

17. Let

$$A = \begin{bmatrix} \alpha & 1 & 0 \\ \beta & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

Find all values of α and β for which

a. A is singular.

b. A is strictly diagonally dominant.

c. A is symmetric.

d. A is positive definite.

Chap 7: Iterative Techniques in Matrix Algebra

7.1

5. The following linear systems $Ax = b$ have x as the actual solution and $\tilde{x}$ as an approximate solution. Compute $\|x - \tilde{x}\|_\infty$ and $\|A\tilde{x} - b\|_\infty$.

   a. $\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63}$.

   $\frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168}$,

   $x = \left(\frac{1}{7}, -\frac{1}{6}\right)^t$,

   $\tilde{x} = (0.142, -0.166)^t$.

7. Show by example that $\| \cdot \|_\infty$, defined by $\|A\|_\infty = \max\limits_{1 \le i, j \le n} |a_{ij}|$, does not define a matrix norm.

13. Prove that if $\| \cdot \|$ is a vector norm on $\mathbb{R}^n$, then $\|A\| = \max_{\|x\|=1} \|Ax\|$ is a matrix norm

7.2

3. Which of the matrices in Exercise 1 are convergent?

a. $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$

e. $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

f. $\begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 7 \end{bmatrix}$

g. $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$

h. $\begin{bmatrix} 3 & 2 & -1 \\ 1 & -2 & 3 \\ 2 & 0 & 4 \end{bmatrix}$

7.3

13. Show that if $\| \cdot \|$ is any natural norm, then $(1/\|A^{-1}\|) \le |\lambda| \le \|A\|$ for any eigenvalue λ of the nonsingular matrix A.

7.4

1. Compute the condition numbers of the following matrices relative to $\| \cdot \|_{\infty}$.

a. $\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$

b. $\begin{bmatrix} 3.9 & 1.6 \\ 6.8 & 2.9 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 \\ 1.00001 & 2 \end{bmatrix}$

d. $\begin{bmatrix} 1.003 & 58.09 \\ 5.550 & 321.8 \end{bmatrix}$

e. $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$

f. $\begin{bmatrix} 0.04 & 0.01 & -0.01 \\ 0.2 & 0.5 & -0.2 \\ 1 & 2 & 4 \end{bmatrix}$

9. Use four-digit rounding arithmetic to compute the inverse H^{-1} of the 3×3 Hilbert matrix H, and then compute $\hat{H} = (H^{-1})^{-1}$. Determine $\|H - \hat{H}\|_{\infty}$.

Chap 8: Approximation Theory

8.1

5. Given the data:

x_i	4.0	4.2	4.5	4.7	5.1	5.5	5.9	6.3	6.8	7.1
y_i	102.56	113.18	130.11	142.05	167.53	195.14	224.87	256.73	299.50	326.72

a. Construct the least squares polynomial of degree 1, and compute the error.

b. Construct the least squares polynomial of degree 2, and compute the error.

c. Construct the least squares polynomial of degree 3, and compute the error.

d. Construct the least squares approximation of the form be^{ax}, and compute the error.

e. Construct the least squares approximation of the form bx^a, and compute the error.

8.2

3. Find the linear least squares polynomial approximation on the interval $[-1, 1]$ for the following functions.

a. $f(x) = x^2 - 2x + 3$

b. $f(x) = x^3$

c. $f(x) = \dfrac{1}{x + 2}$

d. $f(x) = e^x$

e. $f(x) = \dfrac{1}{2}\cos x + \dfrac{1}{3}\sin 2x$

f. $f(x) = \ln(x + 2)$

11. Use the Gram-Schmidt procedure to calculate L_1, L_2, and L_3, where $\{L_0(x), L_1(x), L_2(x), L_3(x)\}$ is an orthogonal set of polynomials on $(0, \infty)$ with respect to the weight functions $w(x) = e^{-x}$ and $L_0(x) \equiv 1$. The polynomials obtained from this procedure are called the **Laguerre polynomials**.

8.3

3. Use the zeros of $\tilde{T}_4$ to construct an interpolating polynomial of degree 3 for the functions in Exercise 1.

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1. Use the zeros of $\tilde{T}_3$ to construct an interpolating polynomial of degree 2 for the following functions on the interval $[-1, 1]$.

  a. $f(x) = e^x$ **b.** $f(x) = \sin x$

  c. $f(x) = \ln(x + 2)$ **d.** $f(x) = x^4$

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7. Find the sixth Maclaurin polynomial for $\sin x$, and use Chebyshev economization to obtain a lesser-degree polynomial approximation while keeping the error less than 0.01 on $[-1, 1]$.

9. Show that for each Chebyshev polynomial $T_n(x)$, we have

$$\int_{-1}^{1} \frac{[T_n(x)]^2}{\sqrt{1 - x^2}} \, dx = \frac{\pi}{2}.$$

Chap 9: Approximating Eigenvalues

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